The Keynesian Multiplier in an Endogenous Credit-Money Economy*

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Abstract. The aim of this paper is to contribute to the theoretical discussion on the Keynesian multiplier, particularly regarding its nexus to endogenous money. Is the multiplier principle consistent with an endogenous credit-money economy? The answer will be given by analyzing the two different versions of the multiplier in Keynes' General Theory. Additionally, the paper reviews Basil Moore's alternative approach, where the income velocity of money replaces the marginal propensity to consume in the multiplier formula. We argue that both the traditional Keynesian multipliers and Moore's multiplier are incomplete. Nonetheless, they can benefit from each other. Their useful properties are integrated to a new version of the multiplier drawing on a time component and the reflux principle of endogenous money.
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1 Introduction

In several writings, Basil Moore (1988, 1994, 2006, 2008) pointed out that two basic principles of Post Keynesian Economics—endogenous credit money and the

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traditional multiplier analysis—are at odds concerning the role and the creation of savings. Moore, as a precursor of the endogenous money approach, therefore declared the "demise of the Keynesian multiplier." Allin Cottrell (1994), in a reply to Moore, defended the multiplier. He argued that Moore left out important adjustment processes and that Moore's conundrum concerning equilibration of investment and saving 'was surely resolved to most people's satisfaction by the 1940s' (Cottrell 1994: 113). Later, Paul Dalziel (1996) joined the debate to clarify Cottrell's points.

The rough statement of this paper is: There are indeed inconsistencies in the standard explanation of the multiplier. Nevertheless, this does not annul the multiplier as such, if it is explained more carefully. It simply points out the need for a sound derivation that is in line with the nature of a credit-money economy. Ironically enough, Moore himself made a proposal, relating the multiplier to the income velocity of money (Moore 2006: 376), yet, the approach was little-noticed in literature. The paper shows that both the standard multiplier and Moore's version are incomplete but can benefit from each other. Both are mere special versions of a more general multiplier that combines their useful properties.

The paper is organized as follows: The next section analyzes the key elements of the different multiplier concepts as deduced from the General Theory. It reviews and extends Moore's critique of these concepts. The third section deals with Moore's alternative multiplier which is related to the income velocity of money. Section four elaborates on leakages in the circuit that are absent in Moore's concept and insufficiently modeled in the standard approach. Section five combines these findings to an integrated version of the multiplier. The final section concludes.

A short note on assumptions: Throughout this paper we consider a closed economy with underemployed resources. Prices are mainly determined by unit costs outside the model; inflation is not a monetary phenomenon, thus we deal with nominal terms only.

2 Two versions of the standard multiplier

Keynes considered the two components of aggregate demand in a closed economy—consumption (C) and investment(I)—to be driven by completely different forces. Investment demand is very volatile and thus the ultimate cause of economic fluctuations:

> 'The theory can be summed up by saying that, given the psychology of the public, the level of output and employment as a whole depends on the amount of investment. I put it in this way, not because this is the only factor on which aggregate output depends, but because it is usual in a complex system to regard as the *causa causans* that factor which

> is most prone to sudden and wide fluctuation. More comprehensively, aggregate output depends on the propensity to hoard, on the policy of the monetary authority as it affects the quantity of money, on the state of confidence concerning the prospective yield of capital-assets, on the propensity to spend and on the social factors which influence the level of the money-wage. But of these several factors it is those which determine the rate of investment which are most unreliable, since it is they which are influenced by our views of the future about which we know so little.'
> (Keynes 1937c: 221)

In contrast, the demand for consumption goods, is given by a rather stable propensity to consume ($c = 1-s$) out of disposable income. The multiplier formula derives from the following identities:

$$Y \equiv C + I \tag{1}$$
$$Y \equiv C + S \tag{2}$$
$$S \equiv sY \tag{3}$$

Substituting (3) into (2) yields

$$C \equiv (1 - s)Y. \tag{4}$$

Substituting (4) into (1) yields

$$Y \equiv \frac{1}{s}I \tag{5}$$
$$\Delta Y \equiv \frac{1}{s}\Delta I \tag{6}$$

Considering ΔI to be exogenously determined and s to be a behavioral parameter makes (6) a behavioral equation

$$\Delta Y = \frac{1}{s}\Delta I \tag{7}$$

where $1/s$ is the investment multiplier, typically denoted as k.

While the multiplier formula itself is quite clear, the process behind it is ambiguous right from the start. There are at least two different versions that have been developed out of Keynes' works: In order to assess Moore's critique, we will separately investigate the textbook version of the multiplier, which we will call the *series multiplier*, and Keynes' so called *logical multiplier* on the other hand.

The series multiplier

The version of the *series multiplier*, that was particularly proclaimed by Hansen (1953: 108), has found its way into nearly every macroeconomic textbook. It describes a chain of events in logical time, where an initial increase in investment in the "first round" generates additional income, which is partly spent for consumption and partly hoarded in the "second round". The spent part induces additional income that, again, is spent and saved in a certain proportion in "round number three", and so on. This 'converging series of ever diminishing waves of expenditures' (Meade 1975: 84) yields:

$$\Delta Y = (1 + c + c^2 + c^3 + ...)\Delta I = \frac{1}{s}\Delta I \quad \text{with} \quad 0 < c, s < 1 \tag{8}$$

The process stops as soon as additional saving equals additional investment. It is the increasing income that adjusts planned saving step by step to planned investment. In other words: The lower the saving rate s, the more income is generated before saving is equal to investment again.

The great success of the series multiplier presumably arises from the underlying familiar process of receipts, spending and saving. This characteristic in accordance with the resulting simple formula gives it a great didactic appeal. Nonetheless, if it is supposed to have practical relevance, the model should coherently abstract from reality. Moore (1988, 1994, 2006, 2008) states that this is not the case for several reasons, which cannot be discussed at length here.[1] We will only focus on one crucial point of the Cottrell-Moore debate, that has not sufficiently been made clear: For Cottrell

> '... "planned" or "intended" (or *ex ante*) I and S are not identical, and must be brought into equality by means of some mechanism or other ...' (Cottrell 1994: 113-4) (original emphases)

In order to defend the series multiplier, this mechanism has to be discovered. It is not convincing to simply refer to the mere technical explanation that comparative static analysis requires a stability condition. So, why should voluntary decisions of investors and savers adjust, such that $S^v = I^v$ (where superscript v denotes "voluntary")? Keynes provides a puzzling answer:

> 'An increment of investment in terms of wage-units cannot occur unless the public *are prepared to* increase their savings in terms of wage-units. Ordinarily speaking, the public will not do this unless their

[1] Particularly, the paper does not intend to discuss whether comparative static modeling by itself is useful or realistic, which is one of Moore's central assaults. Rather, the scope is to work out inconsistencies within the line of argument of the standard multiplier.

> aggregate income in terms of wage-units is increasing. Thus their effort to consume a part of their increased incomes will stimulate output until the new level [...] of incomes provides a margin of saving sufficient to correspond to the increased investment. The multiplier tells us by how much their employment has to be increased to yield an increase in real income sufficient *to induce them to do the necessary extra saving ...*'
> Keynes (1936: 117) (emphases added; S.G.)

The passage points out the role of savings as a means of finance. Following this rationale, savings fund the initial investment, which forces them to be equal. The attempt persists in more recent Keynesian literature. For instance, Kahn (by citing Warming (1932: 215-6)) affirms that

> 'He [Warming; S.G.] pointed out that it is the extra saving made out of increased income which is 'the real source of investment'; 'the secondary employment must continue until the total created income causes so much saving that the original investment can be paid.'
> (Kahn 1984: 101)

Bailly, in his critique of the series multiplier, points out:

> 'In the equation $\Delta Y = \Delta C + \Delta I$, ΔI represents the production of investment goods, *but not their purchase*. The proof of this is that the income multiplication mechanism is assumed to unfold for so long as $\Delta S < \Delta I$, that is, throughout the time when the economy is still without the financial resources for purchasing the investment goods produced.'
> (Bailly 2008: 141) (original emphases)

This explanation is completely un-Keynesian, a recurrence to Say's Law, but in fact it is very common. From that point of view, supply governs demand: At first, someone produces a capital good without facing any effective demand for it. The public still cannot afford it as long as $\Delta S^v < \Delta I^v$. Unlike the public, the producer of the capital good is (fortunately) not constrained by a lack of savings. He pays wages that workers partly save and partly spend on consumption goods. Spending induces further income within the consumption goods sector, which is partly saved and partly spent as well. The public saves exactly as much as required for someone to eventually afford the capital good initially produced. At this point the process interrupts at once.

By trying to find a suitable model for the plausible chain of spending and income generation, the causality of I and S has been turned upside down. The multiplier is considered to explain how investment is made possible by savings, while the core

of the General Theory was meant to explain that investment governs savings.[2] Keynes himself overcame this contradiction in his post-General Theory writings, where he added the *finance motive* to hold cash:

> 'Planned investment—i.e. investment *ex-ante*—may have to secure its "financial provision" *before* the investment takes place; that is to say, before the corresponding saving has taken place. It is, so to speak, as though a particular piece of saving had to be earmarked against a particular piece of investment before either has occurred, before it is known who is going to do the particular piece of saving, and by someone who is not going to do the saving himself. [...] This service may be provided either by the new issue market or by the banks;—which it is, makes no difference. [...] But 'finance' has nothing to do with saving. At the 'financial' stage of the proceedings no net saving has taken place on anyone's part, just as there has been no net investment. 'Finance' and 'commitments to finance' are mere credit and debit book entries, which allow entrepreneurs to go ahead with assurance.'
> (Keynes 1937a: 246-7) (original emphases)

> 'There is, however, no such necessity for individuals to decide, contemporaneously with the investment-decisions of the entrepreneurs, how much of their future income they are going to save. To begin with, they do not know what their incomes are going to be, especially if they arise out of profit. But even if they form some preliminary opinion on the matter, in the first place they are under no necessity to make a definite decision (as the investors have to do), in the second place they do not make it at the same time, and in the third place they most undoubtedly do not, as a rule, deplete their existing cash well ahead of their receiving the incomes out of which they propose to save, so as to oblige the investors with "finance" at the date when the latter require to be arranging it. Finally, even if they were prepared to borrow against their prospective savings, additional cash could not become available in this way except as a result of a change of banking policy. Surely nothing is more certain than that the credit or "finance" required by ex-ante investment is not mainly supplied by ex-ante saving. [...] The ex-ante saver has no cash, but it is cash which the ex-ante investor requires. On the contrary, the finance required during the interregnum between the intention to invest and its achievement is mainly supplied by specialists, in particular by the banks, which organise and manage a revolving fund of liquid finance. For "finance" is essentially a revolving

[2]Cf. Trigg (2003) for a further discussion.

fund. It employs no savings.'
(Keynes 1937b: 664-6)

Following the endogenous money approach (that heavily draws on these articles by Keynes), finance does not stem from saving. Financial resources for investment are usually provided by banks that create credit *ex nihilo*. When a loan is granted, a debt and a deposit occur concurrently at the borrowers' account. Once the borrower spends the money on newly produced capital goods, the producer receives deposits, which can be considered as temporary saving. If they are spent later on, someone else earns and temporarily saves them. Thus, finance creates saving and not the other way round. If finance does not require saving, there is no market constraint for voluntary saving to be on par with investment. So, if you accept the endogenous money approach, then you must reject the notion that saving finances investment.

A second look at the quote of Keynes on page 4 reveals another possible solution. One could link that passage to Keynes' elaborations on the relation between saving and investment (Keynes 1936: 64). In this respect, the multiplier is the mechanism that brings about the necessary ex post identity of both terms. Dealing with this explanation requires a short detour on different meanings of ex ante and ex post investment and saving in economics, because these concepts have been subject to confusion with regard to the multiplier. The puzzle boils down to the following: 'Altho S equals I by definition, Keynes holds, at the same time, that the multiplier makes them equal.' (Lutz 1938: 608) The root of the matter, as will be shown, is the double assignment of ex ante and ex post (i) for national accounts, and (ii) for comparative static modeling. Both concepts make use of these terms on incompatible semantic levels.

(i) In terms of national accounting, where historical time matters, ex post and ex ante are by-words for *actual* and *voluntary*. Ex post (i. e. for any period of time, be it a year or a day) the flow of saving equals the flow of investment in a closed economy:

$$S^a \equiv I^a \tag{9}$$

where the superscript a denotes "actual". These actual (ex post) flows of investment and saving comprise both voluntary (ex ante) and non-voluntary fractions:

$$S^a \equiv S^v + S^{\bar{v}} \quad \text{and} \quad I^a \equiv I^v + I^{\bar{v}} \tag{10}$$

While actual savings must be identical to actual investment, there is no such constraint for voluntary savings and investment. They may be equal or not, and there is neither a tendency, nor a mechanism towards equality. If voluntary investment

exceeds (falls short of) voluntary saving, there will simply be a remainder of non-voluntary saving (non-voluntary investment). This is the way Moore (2006) uses these terms. He rejects the notion of a behavioral saving function. Investment, that is capital accumulation, plays the active part and actual saving is simply the accounting record of investment. Just as loans create deposits, an additional (credit-financed) investment creates additional savings, be they voluntary or non-voluntary. Identity (9) always holds.

(ii) Comparative static analysis focuses on voluntary saving and investment only. Based on logical time, ex ante and ex post mean *before* and *after* the system has adjusted to a shock. This is similar to *out of the old equilibrium* and *back in the new equilibrium*. Applied to the series multiplier: when voluntary investment rises, ex ante investment is higher than ex ante saving:

$$I^v > S^v \tag{11}$$

The system is out of equilibrium. This stirs up an adjustment process of voluntary decisions to spend and save (behavioral saving function), which raises savings until they equal investment ex post. The system strives for equilibrium, where

$$I^v = S^v. \tag{12}$$

Let us put things together again: If our question "Why should the decisions of investors and savers equilibrate?" is answered by "Because they have to be identical ex post!", then we would disregard the semantic differences just pointed out. Essentially it would mean: voluntary saving must adjust to voluntary investment in equilibrium ($S^v = I^v$) because actual saving and actual investment are necessarily identical ($S^a \equiv I^a$). Clearly, this is a mix-up of the two different meanings of ex post. No law-like equality of *voluntary* savings and investment can be derived from the identity of *actual* savings and investment.

And even if that closure were feasible: We still could not determine the length of the adjustment process, since $S \equiv I$ for any considered period—even a single day. Moreover, a contradiction would occur: if we would suppose the multiplier to work out fully within one year, then after half a year we would still have $S < I$, despite $S \equiv I$ applies to half-year accounting as well. Thus, the reasoning that ex post identity arises from equilibration of ex ante terms, turns out misleading.

We should sum up our findings so far: The multiplier process

- does not show how investment is financed through savings.
- does not equilibrate voluntary savings and investment solely because actual savings and investment are identical.

Two common explanations of the multiplier process are rendered void. But in fact, this is not the whole story. Both Cottrell (1994) and Dalziel (1996) proposed alternative explanations. Cottrell draws on Keynes' logical multiplier. Dalziel draws on process analysis of the series multiplier. We will deal with Dalziel's proposal first, to stick to the framework of the series multiplier.

Dalziel joined the Cottrell-Moore debate in 1996. He carefully pointed out how voluntary saving could end up equal to voluntary investment. It all begins with an additional initial investment that generates additional income of the same amount:

$$\Delta I \equiv \Delta Y_0 \tag{13}$$

The additional income of one round $r > 0$ is used for additional consumption and voluntary saving in the next round $r + 1$:

$$\Delta Y_r \equiv \Delta C_{r+1} + \Delta S_{r+1} \tag{14}$$

Additional expenditures on consumption in one round generate additional income of the same amount:

$$\Delta C_r \equiv \Delta Y_r \tag{15}$$

> 'Process analysis makes clear that the result is not some quirk of the underlying mathematics, or a matter of choice about assumed equilibrating mechanisms (interest rate or real income), but is the inevitable outcome of two very simple economic identities: expenditure equals income and income equals consumption plus saving.'
> (Dalziel 1996: 318)

Unfortunately, these identities themselves tell us nothing about how the process will come to an end, even if Dalziel argues the converse:

> 'Hence, it is possible to reject as an empirical matter any behavioral hypothesis about saving decisions without affecting the validity of the multiplier theory.'
> (Dalziel 1996: 326)

This statement is wrong. Equation (14) must be supplemented by a behavioral function that determines the proportion of ΔC_{r+1} to ΔS_{r+1}. Otherwise, voluntary saving could be even negative for some rounds whereby the process might never stop. As Keynes put it:

> 'My theory itself does not require my so-called psychological law as a premise. What the theory shows is that if the psychological law is *not* fulfilled, then we have a condition of complete instability.'
> (Keynes 1973: 276)

Only if saving behavior secures a positive (but not complete) leak out of the circuit the process will eventually yield

$$\Delta I = \Delta S \equiv \sum_{i=1}^{R} \Delta S_i \quad \text{if} \quad \varnothing \Delta S_i > 0. \tag{16}$$

If we accept a positive leakage, represented by a stable stream of saving, then Dalziel (1996) presents a solution that neither mixes up the different ex post concepts nor presumes saving as a means of finance. However, in order to calculate the multiplier two caveats apply, that are central to the remainder of this paper.

1. What happens when moving from "rounds" to concrete time intervals? Is it feasible to define a round as a month, a quarter or a year? Actually, we do not know how long a round takes. Since the multiplier is a dimensionless variable, the length of the adjustment period is undefined. Will it take a month or a decade until a multiplier of, say, 4 has come into effect? The answer can only be given from outside the model which makes identifying concrete multiplier effects arbitrary. Thus, a time dependent multiplier is necessary, which will be subject to section 3.

2. What is the nature of the leakage? Is it soundly represented by savings which are defined as *income not consumed*? In other words: Is it correct to assume that for any round $r > 0$ additional expenditures solely consist of consumption? We will discuss a reformulation of the leakage in section 4.

But for the sake of completeness, we should first turn to Cottrell's suggestion to solve the *I*-*S* conundrum. Cottrell refers to another version of the multiplier, namely, Keynes' logical multiplier.

The logical multiplier

Keynes dropped the process analysis method when he developed his

> '... logical theory of the multiplier, which holds good continuously, without time-lag, at all moments of time [...] in every interval of time the theory of the multiplier holds good in the sense that the increment of aggregate demand is equal to the increment of aggregate investment multiplied by the marginal propensity to consume.'
> (Keynes 1936: 122-3)

The line of argument is as follows: The *logical multiplier* refers to an increase of employment in the capital goods sector that forces the consumption goods sector

to expect an increase in demand on a scale determined by the long run marginal propensity to consume (MPC) out of current income.[3]

The concept is static. It does not explain how savings gradually grow until they equal the change in investment—it does not refer to voluntary savings at all. It simply refers to actual savings that appear simultaneously with investment, whereby the logical multiplier does not mix up those two ex post concepts described above. Moreover, savings are not needed to finance investment, whereby the logical multiplier in no way runs counter to the endogenous money approach (Gnos 2008: 191). Thus, the logical multiplier might remedy the difference in method between endogenous money and the multiplier principle.

But also the logical multiplier does not answer our two questions concerning the nature of the leakage and the length of the time span. These questions are merely skipped within a purely static framework. While the series multiplier assumes that one round equals one period (of undefined length), the logical multiplier assumes that all rounds proceed within one single period (of undefined length). The logical multiplier does not model how the multiplier process unfolds, it simply shows its result. Keynes was aware that real processes take gradual effect and need an interval of time to unfold. Keynes (1936: 122-3) and other authors (Cottrell 1994; Bailly 2008) suppose two causes for this—either consumers do not instantaneously adjust their spending behavior to their new level of income (Robertson-lag), or the producers of consumption goods have not foreseen the increment in demand and need some time to increase production (Lundberg-lag). Yet, the effect is the same:

> '... a temporary departure of the marginal propensity to consume away from its normal value, followed, however, by a gradual return to it. [...] when the deferred consumption is enjoyed, the marginal propensity to consume rises temporarily above its normal level, to compensate for the extent to which it previously fell below it, and eventually returns to its normal level;'
> (Keynes 1936: 123-4)

This is in fact a dynamic adjustment process of the MPC, that can be modeled by a damped (or critically damped) oscillation. As savings instantaneously rise with

[3] There is yet another version of the multiplier, developed by Hartwig (2004, 2008), which closely relates to the logical multiplier. His *structural multiplier* is based on Marxian reproduction schemes in a model with two departments—one producing capital goods, the other producing consumption goods. 'It designates the proportion of department II relative to department I that is necessary for completely successful reproduction and that must be sustained in an expanding economy to ensure the identity between saving and investment' (Hartwig 2008: 23). With the structural multiplier $c/(1-c)$ the entrepreneurs of department II have a guideline on how much to increase their production when they face a certain increase of production in department I. Without underestimating the unique features of Hartwig's approach we may attach it to the logical multiplier for the scope of this paper.

investment, the MPC must initially equal zero to compensate for

$$\Delta Y = \Delta I = \Delta S. \tag{17}$$

Until then, no multiplying process has happened. The multiplier sets in by assuming that the MPC reaches back to its normal or long run level—the average propensity to consume (APC). Moore (1994: 127) doubts whether such a normal level actually exists. According to him, the unpredictable, path-depending future does not allow for assuming a stable value for the MPC. As long term expectations continuously change along the time path, so do equilibrium values. He concludes: 'Ergo, there can be no Keynesian income multiplier' (Moore 1994: 127). However, for the mere existence of the logical multiplier it is irrelevant whether the MPC reaches back to a long run trend or not. Once it eventually exceeds zero, there will be at least *some* multiplying effect. But certainly, nothing is said about the concrete value of the multiplier then. In other words, measuring the income effect will be far more complicated if you drop the assumption that the MPC eventually equals the APC. You have to find another suitable value for s in the multiplier formula (7) then.

But let us, for the sake of the argument, suppose the MPC to be a (critically) damped oscillation that reaches back to the APC. Still is unclear how long it takes the MPC to rebound, which is important to measure the multiplier effect within a certain time span. Basically, a related question concerns the series multiplier, where we do not know how long "one round" actually takes. Both approaches entail a dimensionless multiplier that does not reveal at which point in time it has worked out half or fully. This calls for a time-dependent multiplier which is subject to the next section.

Before proceeding, we should briefly summarize this section: The well-known series multiplier is a comprehensible way to model the process of expenditures and receipts stemming from an initial demand for capital goods. But in order to find a position of rest, usually there is an unsound explanation how the stability condition (voluntary savings equal voluntary investment) comes into being. Either it draws on a mix-up of different meanings of ex ante and ex post, or on a mix-up of savings and finance. When assessing the process more carefully one can overcome these contradictions. However, two other questions come to the fore: 1. How does a dimensionless multiplier apply to real time processes and 2. what are the leakages out of the circuit? Keynes' logical multiplier is supposed to avoid these questions by dropping process analysis. But it turns out that some adjustment process must be employed to account for time lags. If done so, the MPC becomes a (critically) oscillating variable that somehow reaches back to its long run level. Questionable as this may be, it definitely requires a time-dependent multiplicand (as does the series multiplier) to apply to real processes.

3 The velocity multiplier

Moore (1988: 305; 2006: 376; 2008: 125) develops an alternative approach, where the income velocity of money replaces the MPC as the multiplicand. It derives from the Quantity Equation as follows: The whole volume of trading T_t in a certain period of time equals the amount of money that flows from hand to hand. This amount is decomposed by the stock of money M_t^* and its transactions velocity V_t^*. Then the truism holds that

$$T_t \equiv M_t^* V_t^* \tag{18}$$

which resembles the Quantity Equation for the whole volume of trading. By focussing on the amount of money that is used to buy currently produced commodities and services, the standard Quantity Equation comes up:

$$Y_t \equiv M_t V_t \tag{19}$$

A change in aggregate demand (ΔY_{t+1}) comes along with changes in the stock and/or in the velocity of money. Totally differentiating (19) yields

$$\Delta Y_{t+1} \equiv \Delta M_{t+1} V_t + \Delta V_{t+1} M_t + \Delta M_{t+1} \Delta V_{t+1} \tag{20}$$

which is the additional income of period $t+1$ compared to period t. So much for definitions. To predict next period's additional income (ΔY_{t+1}) Moore (2008: 125) assumes the following:

1. The change of velocity of money is expected to be zero ($\Delta V_{t+1} = 0$)
2. Changes in investment spending are totally financed by bank lending ($\Delta I_{t+1} = \Delta L_{t+1}$)
3. Bank lending finances nothing else but investment expenditures ($\Delta L_{t+1} = \Delta M_{t+1}$)

In the strict sense, assumption No. 3 is not homogeneous in dimensions. While ΔL_{t+1} measures a change in a flow comparing two periods, ΔM_{t+1} measures a change in a stock comparing the beginning and the end of one period. To bypass that problem we may relate the change in the flow to the very beginning of the period under consideration. The change in the flow is then treated as if it were a change in a stock. This way, (20) becomes

$$\Delta Y_{t+1} = V_t \Delta I_{t+1} \tag{21}$$

where the income velocity of money for a certain period resembles the Keynesian multiplier. 'In each period the Keynesian multiplier is the income velocity

of money!' (Moore 2008: 126). This conclusion is remarkable. While Moore repeatedly announced the 'demise of the Keynesian multiplier' he develops his own version, which we may call the *velocity multiplier* from now on.[4]

The idea breaks away from Keynes' original reasoning that the amount of investment is the *causa causans* of aggregate demand just because it is its most volatile component. In this sense, Moore doesn't make a distinction between investment and consumption. What makes the difference between I and C is how they are paid for. While C is usually paid out of current or past income, I is mostly financed by newly created bank credit or by drawing on overdraft facilities. Thus, investment drives aggregate demand precisely because it requires additional credit money. In principle, it is the additional amount of circulating credit money that initiates the multiplier effect. And it is the velocity of the additional credit money which accounts for the value of the multiplier. Moore combines the ideas of endogenous money and the multiplier.

An additional amount of money—created by someone who is willing to borrow and by a bank that is willing to lend—induces a succession of incomes and receipts, just like the process of the series multiplier suggests. The crucial difference is: there is no resting point where the system gravitates towards; nothing leaks out of the circuit. Instead of that, the process happens in real time, which allows to measure the laps of the amount of money in the circuit. The question: "How large is the multiplier in a certain period of time?" boils down to: "How often will the additional money go round during that certain period?" The velocity multiplier has an infinite value for an infinite period, but a definite value for a definite period of, say, 4 per year or 1 per quarter. The series multiplier is the antipode: It yields a finite value for an infinite succession of rounds, while it does not define how long one round actually takes.

This is a great advantage of the velocity multiplier. As the income velocity of money has a concrete time dimension, it applies to processes in historical time. Additionally, voluntary savings do not need to adjust to voluntary investment, whereby the model gets rid of a stability condition, without loosing the ability to yield a solution. Savings are not needed to finance investment. Thus, the approach is coherent with the concept of endogenous credit-money. As consumption and investment differ in the way they are usually paid for, but not in general in the way they influence further expenses, an artificial division can be dropped: Within the standard multiplier framework initial investment exclusively causes consumption,

[4]Moore's recourse to the Quantity Equation has been criticized by some authors as "smacking of monetarism" (Rochon 2008: 168). However, he does only use the Quantity *Equation* which is by itself a mere identity. Moore does *neither* refer to the Quantity *Theory* which tries to explain inflation with monetary growth. *Nor* does he refer to Neo-Quantity Theory, where M is determined by the central bank. To him, M is fully determined by creditworthy credit-demand.

which in turn exclusively causes further consumption, and so on. By no means is investment induced by additional aggregate demand; except for the initial change, investment remains constant throughout the process. There is no accelerator effect.[5] However, within the velocity multiplier framework initial investment causes further expenditures which are not limited to consumption, but may also enclose further investment. The approach does not solely refer to the marginal propensity to consume (MPC), but to a broader term that we will call the *marginal propensity to expend* (MPE) from now on.[6] It consists of additional consumption and investment decisions arising from initial demand. Thus, besides the usual multiplier effects on consumption, it covers crowding-in or accelerator effects.

As a very special property of Moore's multiplier, the MPE is bound to unity. Credit money that is not used for consumption does not leak out, but becomes deposits on bank accounts where banks use it for further business. Depositors do *convenience lending* then (Moore 2006: 366). Only the speed of transactions—represented by V_t—limits the multiplication.

However, the approach has at least three distinct weaknesses. The first one concerns the assumption that velocity is stable along the time span of measurement. Moore (2008: 125) states: 'The velocity of money is continually changing. But since it has a unit root, the best estimate of next period's velocity is the current period's velocity, so the expected change in velocity is zero.' Moore uses the very same reasoning for two diametrically opposing assumptions: While rejecting the proposition of a stable MPC over calendar time due to its unit root, he assumes a stable velocity of money just because it has a unit root. Basically, assuming a stable velocity is as binding as assuming a long run stable MPC. This is particularly problematic, since the velocity of money for Moore's multiplier plays exactly the same part as the MPC for the logical multiplier. However, they differ in a crucial detail: While the MPC initially has to fall to zero and rise afterwards to depict the time lag, the velocity of money does not need to oscillate as the time dimension is already enclosed. Additionally, $\Delta V_{t+1} = 0$ refers to a short time horizon only, while the MPC has to be stable in the long run. In conclusion, assuming stability of velocity is less stringent than stability of the MPC. Nevertheless, it remains a questionable assumption.

The second weakness relates to measurement. Figures of the velocity of money are usually taken ex post by dividing a certain money aggregate by income. In this sense, V_t is a mere residual to fulfill the Quantity Equation. Depending on

[5] Of course there are multiplier-accelerator models that employ an investment function as well. But these effects are necessarily left out when dealing with the series multiplier, because they potentially entail disequilibrium solutions. Cf. Hicks (1959) for an early model.

[6] For clarification: MPE is not Moore's terminology but our's. We could have also used the term *marginal propensity to spend*, but the consequential abbreviation MPS might be conflated with the marginal propensity to save.

the money aggregate under consideration, V_t is higher or lower when M_t is defined narrower or broader. If velocity is regarded as a behavioral parameter, it must draw on the income propagation period of people and firms.[7]

The third (and most crucial) weakness concerns the assumption, that the additional volume of credit money ΔM_t is fully kept within the circuit during the whole period under consideration. As already mentioned, the velocity multiplier sets the leakage to $l_t = 0$, whereby the MPE necessarily equals unity:

$$e_t = 1 - l_t = 1 \quad \text{with} \quad l_t = 0. \tag{22}$$

The additional credit money will never leak out of the circuit. Hoarding is just a temporary phenomenon embodied in a certain velocity of money, but the hoards will be used again sooner or later.[8] This is the reason why an endless period implies an infinite velocity multiplier—a doubtful property of the model. In fact, there are leakages in the circuit. They are subject to the next section.

4 The leakages in the circuit

The standard multiplier assigns the role of the leakage to savings which are defined as *current income not consumed*. Saving is considered a flow out of the system that has no other destination than becoming equal to the flow of investment. The usual way to teach this to undergraduates is to equate savings to hoarding. Machlup pointed out the muddle with that term:

> 'Many of the critics of Multiplier theory were not able to interpret the leakage as anything but "hoarding." Accumulation of idle cash balances (and cancelation of bank deposits through debt repayment) was the only answer which these critics had for their query as to the nature of the leakages. This identification of the leakages with hoarding is liable to make full-blooded Keynesians furious. They usually react to it with an explanation of the meaninglessness of the concept of hoarding in the Keynesian language—but they do not tell their misinterpreters "what happens to the leaked-out funds." They confine themselves to the contention that all that matters is the fact that these amounts are not spent on consumption. This answer, in turn, is apt to make their opponents furious. As a matter of fact, it is irrelevant for the immediate effect what the nature of the leakage really is. It is true that it does not make any immediate difference "what happens to the leaked-out funds." But the critics have nevertheless a perfect right to

[7] Cf. Machlup (1939) for a comprehensive discussion of the matter.

[8] Cf. the dynamic model of Andresen (2006) for an application of the velocity multiplier.

know what happens if the funds are not "hoarded".'
(Machlup 1939: 19)

Machlup is right that the mere existence of the standard multiplier is not challenged by the nature of the leakage as long as at least *something* leaks out. But in order to assess the size of the multiplier it is worth dealing with the nature of leakages in more detail.

As the multiplier is related to growth of income, the leakage is a depletion of the amount of credit money that circulates for currently produced commodities, services and assets. Basically, credit money can leave the circuit in two ways:

1. Net-debtors can settle their debt when they receive an inflow of money—the money ceases to exist.

2. Net-creditors can continuously accumulate idle cash balances or deposits without intending to reduce these stocks some day—money is hoarded in an unproductive way.

Let us handle them separately.

(1) Some may complain a mix-up of effective demand and circulating money here. The reason why we are allowed to equalize these terms comes with the so called *reflux principle* (Rochon 2008; Lavoie 1999; Kaldor and Trevithick 1981). Consider the following: Someone takes out a loan in order to finance an additional investment. At the beginning a debt and a deposit occur concurrently at the borrowers' account. Once the borrower spends the money on newly produced capital goods, the producer receives deposits, while the initial borrower is left over with a tangible asset and a debt. The tangible asset, e. g. a machine, produces goods that generate prospective receipts. Once money flows in, the initial borrower (at least partly) uses it to repay the loan. This is a deliberate decision, since debit interest rates usually exceed credit interest rates. As soon as there isn't any further usage for excess credit money, it is repaid (Lavoie 1999: 106). This is the heart of the reflux principle. It constitutes a leak of credit money out of the circuit if there is no further effective demand. The amount of circulating money decreases by debt settlement.

Suppose a slight variation in the way credit is granted: Given there was not an initial loan but an agreement on overdraft facilities between the bank and its customer. Then, the borrower does not have separate accounts for loans and deposits, but only one single account where inflows decrease debt. The reflux mechanism works automatically then.

Suppose another variation, which is actually a generalization: Given the succession of receipts and expenditures encounters any net debtor's account (and not explicitly the original borrower's account). No matter who is the debtor receiving

the money, she will deliberately or automatically use (at least some part of) the receipts to settle her debt. This points out, that the reflux is not tied to the initial loan but applies to the general amount of credit money in the economy.

Suppose yet another variation, where the circulating money is used by someone to buy newly issued bonds. The bond issuer may take the receipts to deliberately or automatically settle her debt to the bank. In this case, buying newly issued bonds refinances an outstanding credit, which lowers the amount of credit money as well.[9]

All these kinds of leakages pass through the channel of the reflux principle. The money finally ceases to exist. A related channel was suggested by Kahn (1931: 176): The government could be 'saving on the dole' if the initial investment, be it public or private, provides additional employment. Technically speaking, this is an *automatic stabilizer* that reduces the net effect of an additional investment. Disposable income doesn't instantly rise as much as the initial expenditure, since another expenditure drops simultaneously. This kind of leakage is the automatic reduction of a demand component (such as government spending) if another demand component (such as private investment) has risen beforehand. By automatic stabilizers the money ceases to exist right from the start.

Above we raised the question whether the leakage is correctly represented by savings, defined as income not consumed. Being aware of the reflux principle, the answer is 'No!' The leakage should be defined as *money that ceases to exist* which includes buying newly issued bonds and settling debt as well. Consequently, the multiplier *does not show*, how an initial *investment* is financed or paid by savings. What it *does show* is the process until an additional amount of credit money is *re*financed or *re*paid. This is more than a simple renaming. It allows for a crucial understanding of the multiplier process in a credit-money economy. As long as the money is kept within the circuit it is used for additional effective demand. Once it is used by anyone to refinance or repay debt, it leaks out. When the amount of debt returns to its former level the multiplier process has come to its end.

(2) When net creditors accumulate idle cash balances or deposits without intending to reduce these stocks some day, the velocity of that certain amount of money reaches zero, reducing the average velocity of money. However, the money still exists and should not be considered a leakage in the narrower sense. These hoards do not definitely leak out. Banks may use customers' deposits to do further business such as non-proprietary trading. Depositors do *convenience lending* as Moore (2006: 366) has put it. This income has not been consumed but it may induce further demand anyway. That is why leakages should not be confounded with the usual definition of hoardings and savings. Money leaks out when it ceases

[9]The example cautions us to be careful concerning our definition of money. The bonds may act as a means of payment for some transactions as well, albeit they lack general acceptance.

to exist; not when it slows down. The former we will call a leakage; the latter we will call hoarding.

Nonetheless, a slowdown of the income velocity of money (hoarding) has important repercussions on income as Moore's velocity multiplier has shown. Deceleration has become increasingly relevant in the era of financialisation. When deposits buy financial assets that already exist, society plays a zero sum game. Transaction velocity is extremely high within that financial sphere but as long as the money is lapping there, it slows down the income velocity in the productive sphere. So even though hoarding is not a leakage in our framework it is relevant to the multiplier as the next section will show.[10]

5 An integrated multiplier

We may now combine our findings of the last two sections to formulate a new version of the multiplier, which is both time dependent and allows for leakages. There are two important channels:

1. The velocity of money that buys newly produced goods and assets (V_t).
2. The magnitude of the leakage (l_t) within a certain period as determined by $l_t = 1 - e_t$, where e_t is the marginal propensity to expend.

The higher the velocity of money the higher is the multiplier. The more intense the leakage (the lower the MPE) the lower is the multiplier. Both determinants are related to time, and so must be the multiplier. As explained above, the multiplier process begins with an additional amount of credit money at the beginning of period $t + 1$. Drawing on both the leakage and the velocity channel, this period's additional income would yield

$$\Delta Y_{t+1} = V_t(1 - l_t)\Delta M_{t+1} = V_t e_t \Delta M_{t+1} \tag{23}$$

The formula is indeed an integrated or general version of the series and the velocity multiplier. In other words: both the series multiplier and the velocity multiplier can be derived from (23) by making special assumptions.

The velocity multiplier is a special case of the integrated multiplier by the following assumptions: As mentioned in section 3, it sets a leakage of $l_t = 0$ and equates

[10] Of course, financialisation has another implication—money fading *from* the productive *to* the financial sphere. The net inflow to the financial sphere has been largely positive throughout the last decades as the literature on financialisation has pointed out (Palley 2008). Some have called this process a leakage, too. But when deposits buy newly created financial assets from banks, indeed they yield an increase in income, even though there is only a negligible increase of employment. Strictly speaking, there is a multiplication in the income dimension, but in fact a loss in transmission to the labor market dimension.

additional credit demand to additional investment ($\Delta M_{t+1} = \Delta I_{t+1}$). The initial investment is put in place at the very beginning of the period under consideration to ensure dimensional homogeneity. Then, equation (23) becomes

$$\Delta Y_{t+1} = V_t \Delta I_{t+1} \tag{24}$$

which equals equation (21).

Also the series multiplier can be derived from (23). It assigns the role of the leakage to savings ($l_t = s$), and the role of additional autonomous demand to investment ($\Delta M_{t+1} = \Delta I_t$) of the initial round. Furthermore, it defines one round as one period, whereby the velocity of money is set to one per period ($V_t = 1/t$). This yields

$$\Delta Y_{t+1} = (1 - s)\Delta I_t = c\Delta I_t \tag{25}$$

which is the additional income generated by consumption expenditures in the round succeeding the initial round.

Both special cases make implausible assumptions concerning velocity or the leakage. Thus, they should be replaced by the general version displayed by equation (23), which has very useful properties: Depending on the values and the relation of parameters V_t and l_t the integrated multiplier does not necessarily converge to a finite value and it does not necessarily end up infinite for an infinite period of time. Stability is assured, if velocity is sufficiently small and the leakage is sufficiently large, such that

$$V_t(1 - l_t) < 1/t \quad \forall t. \tag{26}$$

However, the multiplicative effect is calculable for any period of time. Thus, the integrated multiplier combines the useful properties of the special versions and avoids their questionable properties.

In addition it shows, that the multiplier is a rather loose concept than a mechanical process. The reflux may occur automatically, but that does not necessarily stop the process. To put it more precisely: Even if an inflow mechanically lowers debt, it may entail demanding a new credit in turn. In mathematical terms: The MPE is not constrained to values between zero and unity. In turn, since $l_t = 1 - e_t$, the leakage is not constrained to values between zero and unity, too. A dimensionless comparative static multiplier needs such an assumption as a stability condition. A time-dependent dynamic multiplier does not. An MPE larger than unity (a negative leakage) is conceivable when credit creation induces further credit creation. Accelerator effects would occur. Likewise effects may stem from an acceleration of the velocity of money. We may refer to these variations as *horizontal*, since they refer to a concrete time span, such as a certain phase of the business cycle. In

other words: The very same amount of credit creation may have different effects in upturns and downturns as they provide different environments, which would imply a 'regime dependent multiplier' (Mittnik and Semmler 2010).

We may consider *vertical* variations as well. They relate to the efficiency of different measures with the same initial credit impulse at the same time. Employing the impulse-response method to a certain increase in the amount of circulating money may help to estimate its effect. However, it assigns average values of the whole economy to the parameters of a certain project.[11] That may not be perfectly wrong, but definitely it is far from being perfectly correct.

As the last section has shown, the leakage is not tied to the initial loan but generally concerns a change in the level of circulating credit money. An increasing level of credit money provides the basis for a higher level of income. In contrast, deleveraging may drag down the level of economic activity. In this sense the multiplier relates to a more general discussion on credit cycles and their influence on the business cycle (Fisher 1933; Biggs et al. 2009; Keen 2010).

Until now, the velocity of money V_t and the leakage l_t (or the MPE e_t) have been treated as parameters. A further step for setting up a more complete model of income generation could be to endogenize these terms. Both the MPE and the velocity of money are still rather vague concepts. A next step should be to find determinants of V_t and e_t. As they are subject to fluctuations along the business cycle, and have by themselves an influence on aggregate income, multiplier analysis resembles dynamic business cycle analysis in this respect.

6 Conclusion

The first part of this paper has clarified and extended Basil Moore's reasoning concerning the demise of the Keynesian multiplier. Even though we don't find the multiplier to demise, Basil Moore has pointed out the shortcomings in common explanations of the multiplier process. The Cottrell-Moore debate has revealed the need for a sound explanation of how the multiplier works. Process analysis helps, but it uncovers two further questions that must be answered to apply the multiplier to real processes:

1. How long does one round of the process and the whole process actually take?
2. What are the leakages in the circuit and where does the money go?

Basically, Moore's velocity multiplier provides an answer to the first question. Even if money velocity is a rather vague concept, it renders a time-dependent multiplier

[11] The very same applies to the standard multiplier, where the APC is usually taken to show the outcome of a certain project.

possible. However, the velocity multiplier disregards the second question. It simply abstracts from any leakages. Section 4 has proven that this assumption is not universally valid. There are indeed leakages in the circuit. The reflux principle relates the leakage in a credit money economy to the settlement of debt. The additional credit money ceases to exist. In a narrow interpretation the multiplier unfolds until an initial loan is refinanced or repaid. But this view only takes a micro-perspective. It loses sight of the overall stock of credit-money. In a broad interpretation (or better: on a macro-level) the multiplier works from take-off to landing at a certain level of credit-money for the whole economy. This is how credit cycles and the business cycle intertwine with the multiplier.

The multiplier is not a mechanical input-output relation (determined by a fixed MPC) but an inducement of further demand, be it consumption or investment. When analyzing the circuit of effective demand and income it seems arbitrary to concentrate on the MPC and to put aside the marginal propensity to invest. They both find their way into the so called marginal propensity to expend (MPE), covering induced consumption and investment after an initial spending. The MPE is the counterpart of the leakage. An MPE larger than unity—which comes with a negative leakage—is possible when accelerator or crowding-in effects occur.

The discussion has shown the two main channels of influence—the velocity of money V_t and the magnitude of the leakage l_t. They form a version of the multiplier that integrates both Moore's velocity multiplier and the usual series multiplier. This integrated multiplier can be applied to processes in real time. Additionally, it sets the basis for business cycle analysis as connected to credit cycles.

V_t and l_t by themselves depend on more detailed determinants, such as animal spirits, bank behavior, liquidity preference, central bank reactions and so on. To put it with Machlup's words:

> 'The theory of the Multiplier, if it is to be of use to those who wish to know the possible and the probable effects of public works, must renounce the attractive appearance of neatness and preciseness. The two variables which seem to play the main parts in the play of the Multiplier must be decomposed into the all too large number of variables which play the important roles in the real world.'
> (Machlup 1939: 27)

Modeling these relations will be subject to future research.

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